Report No. 126

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF MINES
HELIUM ACTIVITY
HELIUM RESEARCH CENTER

INTERNAL REPORT

ELASTIC PRESSURE DISTORTION OF THE VOLUMES OF A 1000 ATMOSPHERE BURNETT COMPRESSIBILITY APPARATUS OVER THE TEMPERATURE RANGE 0° TO 75° C

BY

Ted C. Briggs

Alvin R. Howard

BRANCH Fundamental

PROJECT NO. 7005

DATE October 1969

AMARILLO, TEXAS

Report No. 126

HELIUM RESEARCH CENTER
INTERNAL REPORT

ELASTIC PRESSURE DISTORTION OF THE VOLUMES OF A 1000 ATMOSPHERE BURNETT COMPRESSIBILITY APPARATUS OVER THE TEMPERATURE RANGE 0° TO 75° C

By

Ted C. Briggs and Alvin R. Howard

Branch of Fundamental Research

Project 7005

October 1969

CONTENTS

ILLUSTRATIONS

TABLES

ELASTIC PRESSURE DISTORTION OF THE VOLUMES OF A 1000 ATMOSPHERE BURNETT COMPRESSIBILITY APPARATUS OVER THE TEMPERATURE RANGE 0° TO 75° C

by

Ted C. Briggs[1/] and Alvin R. Howard[1/]

ABSTRACT

A removable-jacket distortion apparatus was constructed and used to measure distortion coefficients for two high-pressure vessels. The measured distortion coefficients were used to compute distortion coefficients for volumes V_1 and $(V_1 + V_2)$ of a 1000 atmosphere Burnett compressibility apparatus for the temperature range 0° to 75° C.

Young's modulus for Armco 17-4 PH stainless steel, heat treated to condition H1150-M, was computed from experimentally determined distortion coefficients. A 10 to 14 percent correction to the values obtained for Young's modulus may be required because pressure vessel end effects were neglected.

The distortion coeffcicents of the compressibility apparatus are believed to be accurate to about one percent.

1/ Research chemist, Helium Research Center, Bureau of Mines, Amarillo, Tex.

INTRODUCTION

The Bureau of Mines Helium Research Center obtains gas phase compressibility data by the Burnett (9)[2/] method. The isothermal

---

2/ Underlined numbers in parentheses refer to items in the list of references at the end of this report.

---

volume of the pressure vessels is a function of the internal and external pressures. For maximum accuracy, a correction must be applied for the distortion due to pressure.

Neglecting the correction of pressure distortion would introduce an error of about 0.15 percent into the calculated compressibility factor for helium at 1000 atmospheres and 0° C.

Burnett (9) used jacketed pressure vessels to reduce the magnitude of the pressure distortion. Subjecting a thick wall cylinder to equal external and internal pressures reduces in magnitude, but does not eliminate, the distortion. Mueller (13), Canfield (10), Blancett (5), and others made distortion corrections to Burnett volumes by using elastic distortion theory and literature values for the required elastic properties.

Briggs and Barieau (7) devised an experiment and procedure to measure external-pressure distortion coefficients and to compute internal-pressure distortion coefficients and Young's modulus from the measured quantities. We use their method to evaluate the elastic pressure-distortion corrections for a newly constructed 1000 atmosphere Burnett type compressibility apparatus.

ACKNOWLEDGMENT

The authors thank the staff of the Branch of Automatic Data Processing for a linear least squares evaluation of $d\ln P_r/dP_{j\,r}$ and computation of average $d\ln Z_r/d\ln P_r$ for each set of experimental data.

EXPERIMENTAL APPARATUS AND EXPERIMENTAL PROCEDURE

The objective of this work is to evaluate the distortion corrections for a specific compressibility apparatus. The apparatus volumes consist of two high-pressure vessels designated as V_{b1} and V_{b2}, the lower chamber of a differential pressure cell, valves, fittings, and connecting tubing. The bulk of the gas is confined in volume V_{b1} or $(V_{b1}+V_{b2})$; therefore, distortion of these volumes is of primary concern. Figure 1 shows the component volumes of the assembled

---

FIGURE 1. - Pressure Containers, Valves, and Fittings of a Burnett Type Compressibility Apparatus.

Burnett apparatus while figures 2 and 3 show design details of the

---

FIGURE 2. - Pressure Container, V_{b1}.

FIGURE 3. - Pressure Container, V_{b2}.

---

vessels V_{b1} and V_{b2}.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

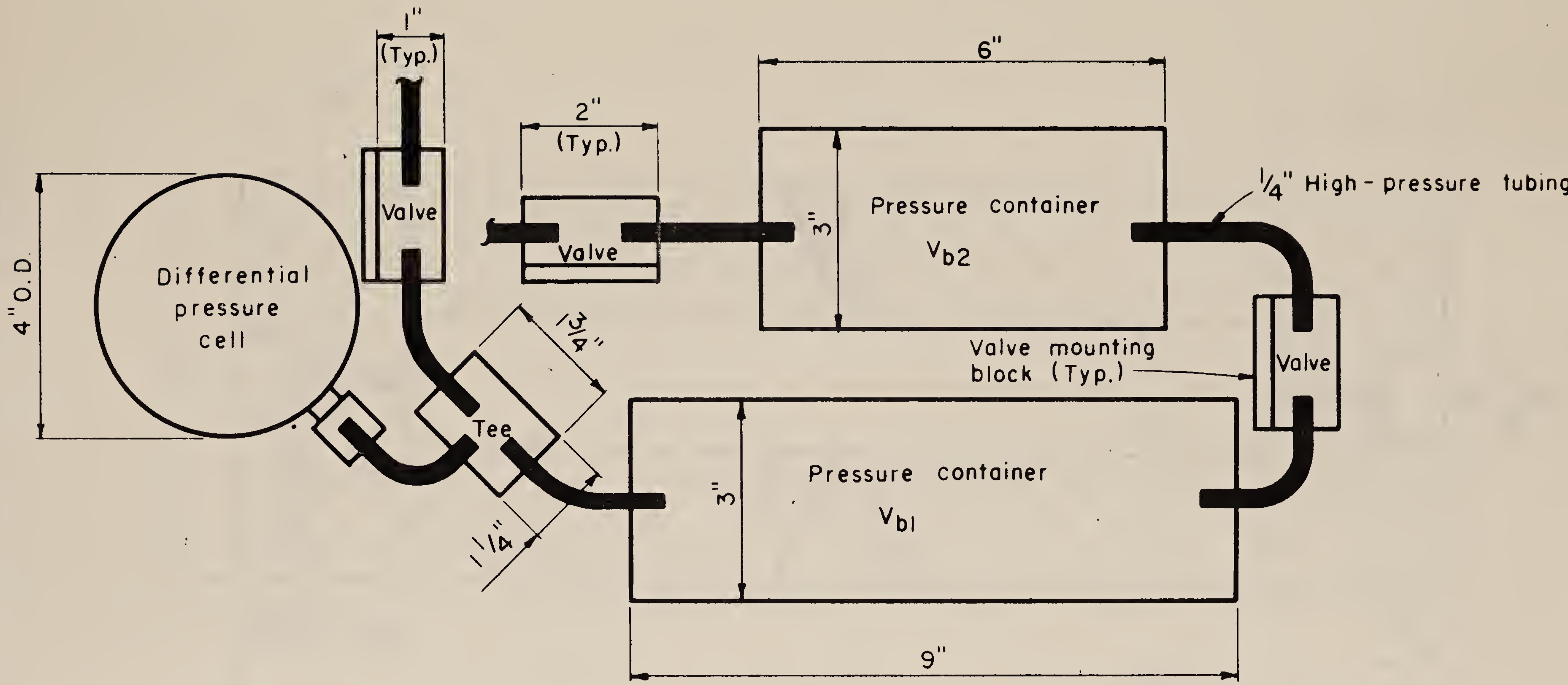

FIGURE 1.- Pressure Containers, Valves, and Fittings of a Burnett Type Compressibility Apparatus.

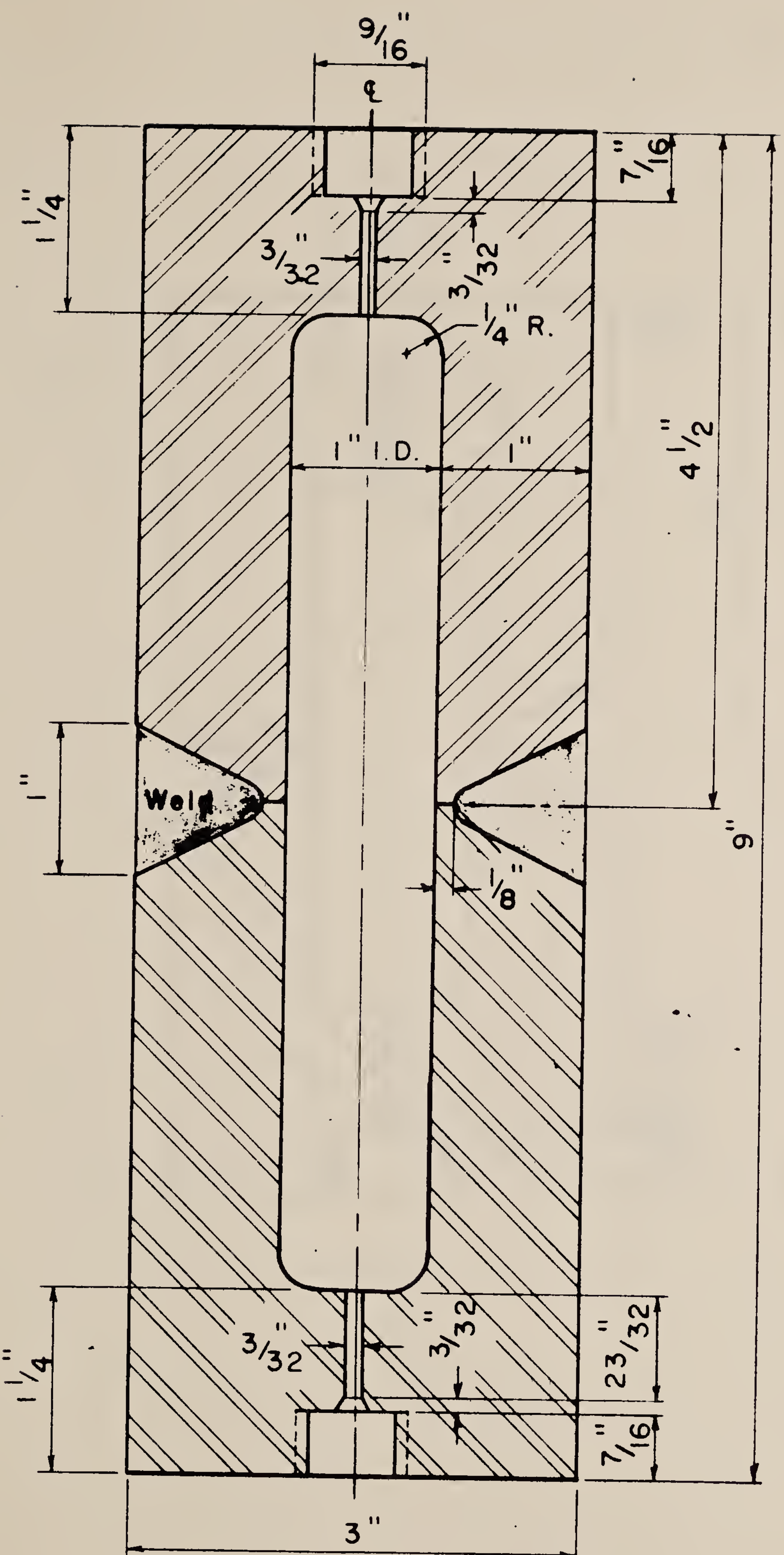

FIGURE 2. – Pressure Container, V_{b1}.

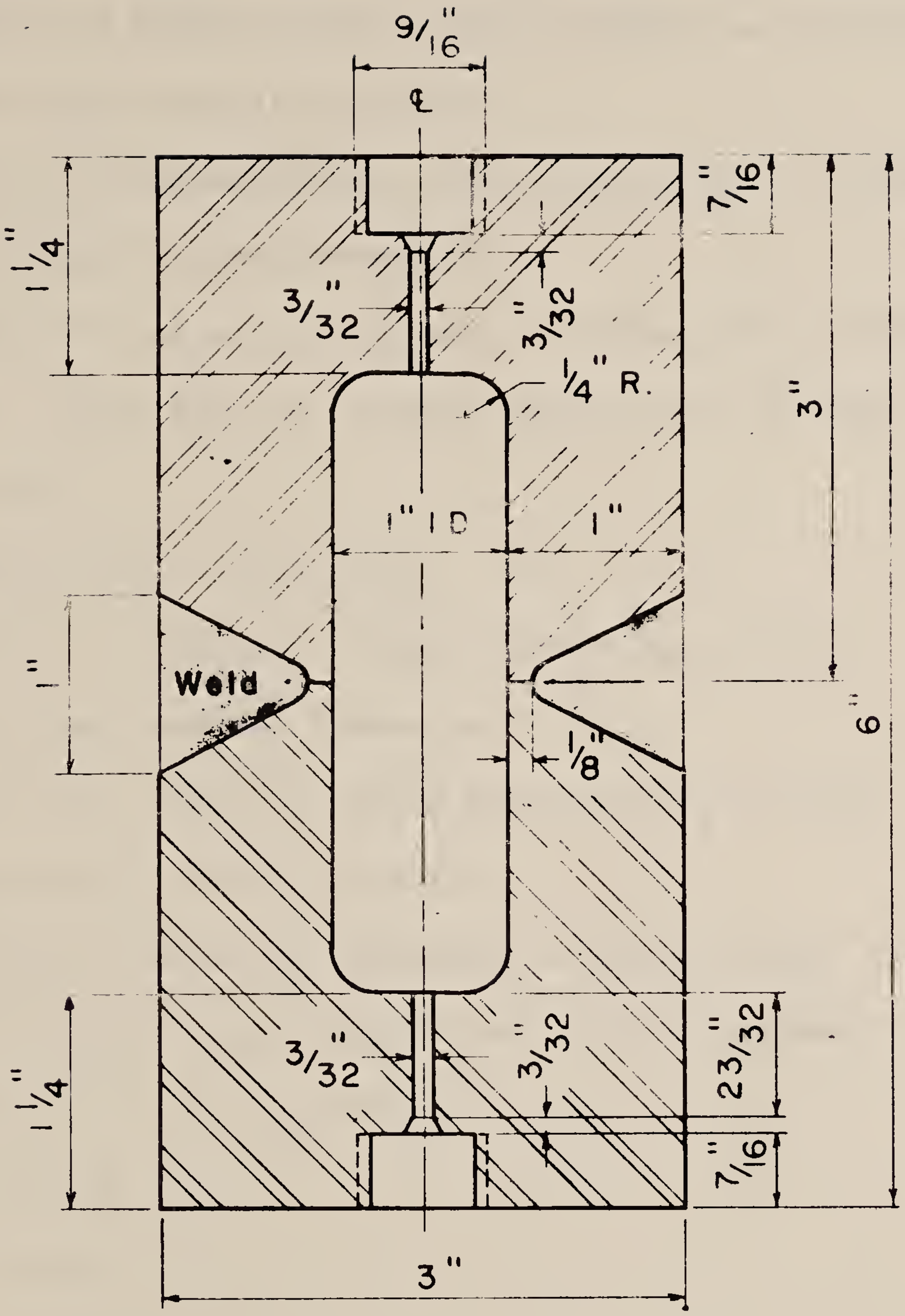

FIGURE 3.- Pressure Container, V_{b2}.

Relevant volumes are listed below and are estimated from the component dimensions.

V_{b1}^{o} = 4.8859 in^3 = volume of the pressure vessel V_{b1} at zero internal and external pressures.

V_{t1}^{o} = 0.0717 in^3 = volume of the tubing portion of V_1 at zero internal and external pressures.

V_{f1}^{o} = 0.0700 in^3 = volume of fittings, including DPI cell and valves, connected to V_1 at zero internal and external pressures.

V_1^{o} = 5.0276 in^3 = $V_{b1}^{o} + V_{t1}^{o} + V_{f1}^{o}$.

V_{b2}^{o} = 2.5297 in^3 = volume of pressure vessel V_{b2} at zero internal and external pressures.

V_{t2}^{o} = 0.0325 in^3 = volume of tubing portion of V_2 at zero internal and external pressures.

V_{f2}^{o} = 0.0176 in^3 = volume of fittings, including valves, connected to V_2 at zero internal and external pressures.

V_2^{o} = 2.5798 in^3 = $V_{b2}^{o} + V_{t2}^{o} + V_{f2}^{o}$.

$(V_{b1}^{o} + V_{b2}^{o})$ = 7.4156 in^3.

$(V_{t1}^{o} + V_{t2}^{o})$ = 0.1042 in^3.

$(V_{f1}^{o} + V_{f2}^{o})$ = 0.0876 in^3.

$(V_1^{o} + V_2^{o})$ = 7.6074 in^3.

The experimental distortion determination method of Briggs and Barieau (7) requires jacketed pressure vessels such that the change of the internal pressure can be determined as a function of changing jacket pressure. Jacketed pressure containers for a 1000 atmosphere Burnett apparatus would have the disadvantage of resulting in rather massive vessels for a relatively small internal volume, particularly if the jackets are adequately designed for equal internal and external pressures.

A removable jacket was purchased for the high-pressure containers specifically for the distortion experiment. The removable jacket was designed so that either volume V_{b1} or V_{b2} could be placed in the jacket. A sketch of the removable jacket is included as figure 4.

FIGURE 4. - Removable Pressure Jacket.

The removable jacket and volume V_{b1} or V_{b2} were placed in a constant temperature bath. The space between the removable jacket and external wall of V_{b1} or V_{b2} was oil filled and was connected to an oil displacement pump and oil filled Bourdon tube pressure gage. The pressure around the vessel could be varied up to the maximum working pressure (10×10^3 psi) of the jacket.

The Bourdon tube gage had a pressure range of 10×10^3 psi and 10 psi scale divisions.

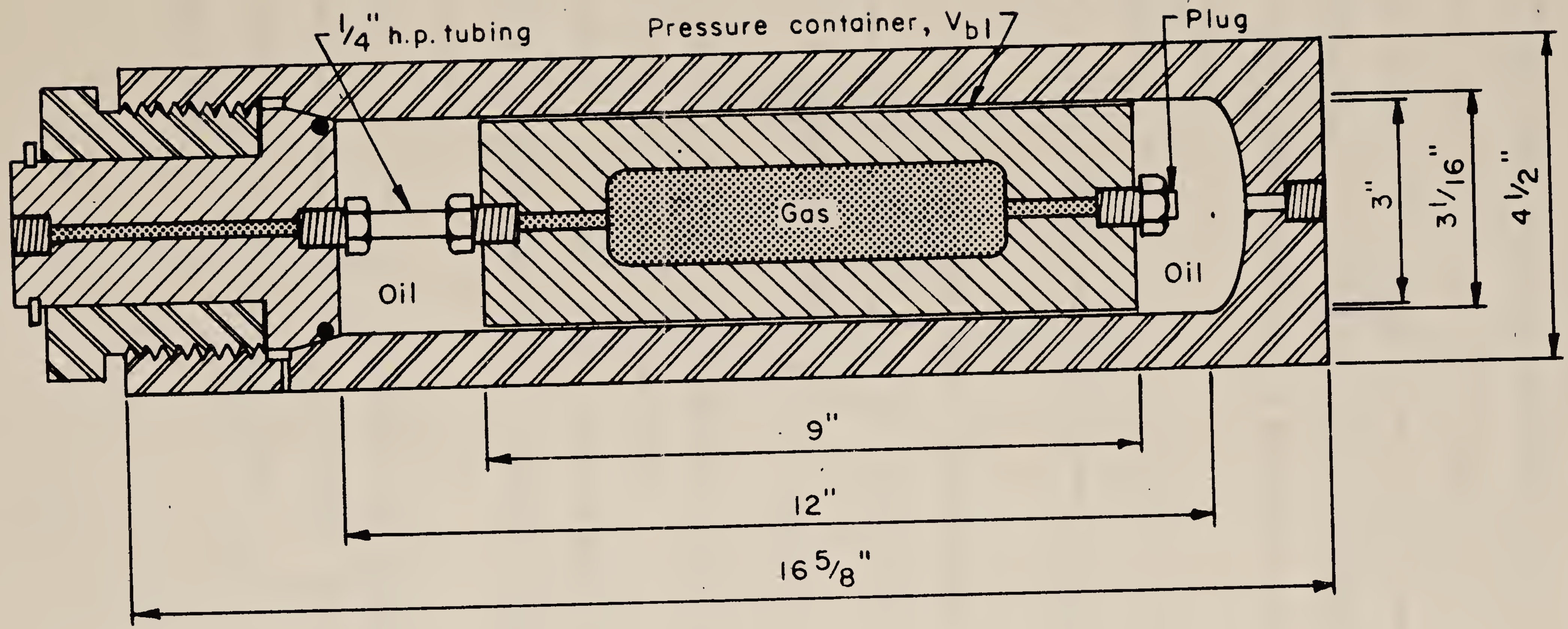

FIGURE 4. – Removable Pressure Jacket.

The inner pressure vessel (V_{b1} or V_{b2}) was connected to a high-pressure (20×10^3 psi) diaphragm-type compressor and to the gas side of a commercial diaphragm differential pressure cell. The reference side of the differential pressure cell was connected to an oil-lubricated piston gage. The piston gage could measure pressures over the range 2 to 800 atmospheres with a precision and accuracy of better than 0.01 percent.

This arrangement allowed the inner vessel to be filled to high pressure, and the pressure could then be measured quite accurately with the piston gage.

A drawing of the distortion apparatus is designated figure 5.

FIGURE 5. - Pressure Distortion Apparatus.

Relevant volumes of the distortion apparatus are listed below and are estimated from the component dimensions.

$V^{o}_{td,uj}$ = 0.0704 in^3 = volume of unjacketed tubing portion of distortion apparatus at zero internal and external pressure.

$V^{o}_{td,j}$ = 0.0135 in^3 = volume of jacketed tubing portion of distortion apparatus at zero internal and external pressure (nipple connecting volume V_{b1} or V_{b2} to jacket cap).

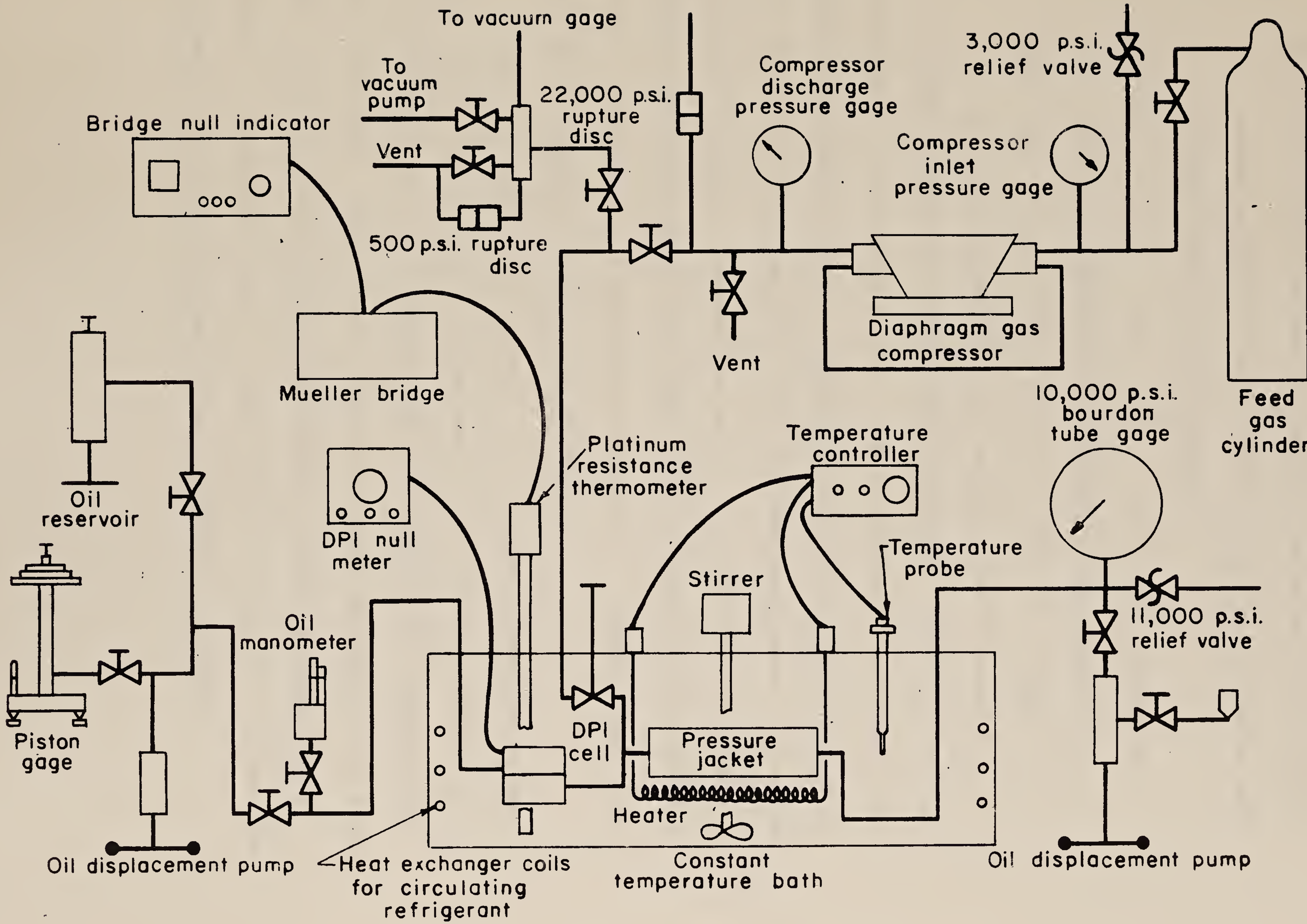

FIGURE 5. – Pressure Distortion Apparatus.

V^o_{fd} = 0.0873 in^3 = volume of fittings connected to distortion apparatus plus volume of hole through jacket cap at zero internal and external pressures.

$V^o_{d1} = V^o_{b1} + V^o_{td,uJ} + V^o_{td,J} + V^o_{fd}$ = 5.0571 in^3 = volume of distortion apparatus when assembled with vessel V_{b1}.

$V^o_{d2} = V^o_{b2} + V^o_{td,uJ} + V^o_{td,J} + V^o_{fd}$ = 2.7009 in^3 = volume of distortion apparatus when assembled with vessel V_{b2}.

The experimental procedure was as follows. Vessel V_{b1} or V_{b2} was placed in the removable jacket and the assembly was placed in a constant temperature bath. Temperature of the bath was adjusted to the desired value as measured with a platinum resistance thermometer and Mueller bridge. Temperatures are in terms of the 1948 International Practical Temperature Scale (IPTS-48) and are the reported nominal values within a precision of $\pm 0.005°$ C. Temperatures in the bath were constant to better than $\pm 0.005°$ C.

The inner chamber of vessel V_{b1} or V_{b2} was filled with helium gas to an initial pressure. Time was allowed for the confined helium to reach temperature equilibrium and the pressure was measured with the piston gage. Resolution of the piston gage was equal to or better than 0.0007 atm at all measured pressures. Jacket pressure was increased in incremental amounts, and each time the jacket pressure was increased, the internal pressure was accurately remeasured.

The differential pressure cell was zeroed with atmospheric pressure applied to both sides of the diaphragm before each run. A correction was applied to the measured pressures for zero shift of the diaphragm as a function of pressure. Zero shift is not very significant during a run as the measured internal pressure changes by about 0.7 atmospheres for a 600 atmosphere change in the external pressure.

Volume V_{b1} or V_{b2} was filled with helium to different pressures for each run. Impurities in the helium totaled less than 25 ppm in all cases.

Runs were made at 0°, 25°, 50°, and 75° C. Experimental observations are recorded in table 1 for vessel V_{b1} enclosed in the pressure jacket, and in table 2 for vessel V_{b2} enclosed in the pressure jacket. P_{jr} and P_r denote jacket pressure and internal pressure, respectively.

TREATMENT OF THE EXPERIMENTAL OBSERVATIONS

Equations for the elastic distortion of a thick wall cylinder are reported in the literature (3, 11, 12, 14, and 7).

Equation 1 describes the pressure distortion

$$\frac{\Delta V}{V^o} = \frac{3(1-2\sigma)R_r^2 + 2(1+\sigma)R_j^2}{E(R_j^2-R_r^2)} P_r - \frac{(5-4\sigma)R_j^2}{E(R_j^2-R_r^2)} P_{jr} , \qquad (1)$$

TABLE 1. - Experimental external-pressure distortion coefficient data, volume V_{b1} + volume V_{fd} + volume V_{td}

0° C

Run No. (17-4-V1)-0-1		Run No. (17-4-V1)-0-2		Run No. (17-4-V1)-0-3	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	504.1649	1	430.9153	1	496.9071
102	504.2972	102	431.0203	102	497.0317
204	504.4246	204	431.1305	204	497.1566
306	504.5543	306	431.2367	306	497.2774
408	504.6795	408	431.3353	408	497.4101
510	504.8169	510	431.4536	510	497.5324
612	504.9320	612	431.5536	612	497.6589

Run No. (17-4-V1)-0-4		Run No. (17-4-V1)-0-5		Run No. (17-4-V1)-0-6	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	365.0349	1	374.9599	1	142.2911
102	365.1256	102	375.0572	102	142.3259
204	365.2124	204	375.1459	204	142.3559
306	365.2981	306	375.2357	306	142.3866
408	365.3865	408	375.3249	408	142.4174
510	365.4751	510	375.4174	510	142.4494
612	365.5621	612	375.5075	612	142.4792

TABLE 1. - Experimental external-pressure distortion coefficient data, volume V_{bi} + volume V_{fd} + volume V_{td}--Continued

25° C

Run No. (17-4-V1)-25-1		Run No. (17-4-V1)-25-2		Run No. (17-4-V1)-25-3	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	364.3717	1	435.1184	1	500.2051
102	364.4592	102	435.2220	102	500.3301
204	364.5437	204	435.3299	204	500.4586
306	364.6302	306	435.4344	306	500.5831
408	364.7166	408	435.5400	408	500.7065
510	364.8028	510	435.6481	510	500.8304
612	364.8898	612	435.7550	612	500.9548

Run No. (17-4-V1)-25-4		Run No. (17-4-V1)-25-5	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	379.8105	1	373.4718
102	379.9024	102	373.5631
204	379.9932	204	373.6494
306	380.0859	306	373.7386
408	380.1769	408	373.8280
510	380.2674	510	373.9158
612	380.3583	612	374.0045

TABLE 1. - Experimental external-pressure distortion coefficient data, volume V_{bl} + volume V_{fd} + volume V_{td}--Continued

50° C

Run No. (17-4-V1)-50-1		Run No. (17-4-V1)-50-2		Run No. (17-4-V1)-50-3	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	368.1443	1	437.4636	1	497.8509
102	368.2326	102	437.5725	102	497.9726
204	368.3202	204	437.6789	204	498.0936
306	368.4071	306	437.7829	306	498.2177
408	368.4941	408	437.8889	408	498.3376
510	368.5803	510	437.9951	510	498.4597
612	368.6676	612	438.0996	612	498.5826

Run No. (17-4-V1)-50-4		Run No. (17-4-V1)-50-5	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	434.0176	1	367.7199
102	434.1211	102	367.8017
204	434.2260	204	367.8885
306	434.3290	306	367.9750
408	434.4336	408	368.0606
510	434.5400	510	368.1471
612	434.6445	612	368.2340

75° C

Run No. (17-4-V1)-75-1		Run No. (17-4-V1)-75-2		Run No. (17-4-V1)-75-3	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	373.9547	1	437.1291	1	494.4104
102	374.0763	102	437.2558	102	494.5310
204	374.1687	204	437.3626	204	494.6457
306	374.2566	306	437.4660	306	494.7700
408	374.3146	408	437.5689	408	494.8910
510	374.3988	510	437.6749	510	495.0127
612	374.4898	612	437.7793	612	495.1281

TABLE 2. - Experimental external-pressure distortion coefficient data, volume V_{b2} + volume V_{fd} + volume V_{td}

0° C

Run No. (17-4-V2)-0-1		Run No. (17-4-V2)-0-2		Run No. (17-4-V2)-0-3	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	366.7151	1	432.9269	1	507.6880
102	366.8023	102	433.0299	102	507.8120
204	366.8849	204	433.1288	204	507.9330
306	366.9681	306	433.2329	306	508.0553
408	367.0508	408	433.3317	408	508.1744
510	367.1364	510	433.4341	510	508.2941
612	367.2233	612	433.5337	612	508.4152

25° C

Run No. (17-4-V2)-25-1		Run No. (17-4-V2)-25-2		Run No. (17-4-V2)-25-3	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	371.1763	1	440.4562	1	506.5533
102	371.2605	102	440.5561	102	506.6707
204	371.3445	204	440.6549	204	506.7901
306	371.4255	306	440.7586	306	506.9100
408	371.5059	408	440.8590	408	507.0287
510	371.5904	510	440.9605	510	507.1470
612	371.6745	612	441.0600	612	507.2662

TABLE 2. - Experimental external-pressure distortion coefficient data, volume V_{b2} + volume V_{fd} + volume V_{td}--Continued

50° C

Run No. (17-4-V2)-50-1		Run No. (17-4-V2)-50-2		Run No. (17-4-V2)-50-3	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	370.4306	1	439.1686	1	510.1689
102	370.5174	102	439.2687	102	510.2893
204	370.5985	204	439.3700	204	510.4096
306	370.6830	306	439.4710	306	510.5292
408	370.7653	408	439.5696	408	510.6524
510	370.8486	510	439.6712	510	510.7740
612	370.9318	612	439.7701	612	510.8905

75° C

Run No. (17-4-V2)-75-1		Run No. (17-4-V2)-75-2		Run No. (17-4-V2)-75-3	
P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm	P_{jr}, atm	P_r, atm
1	370.0747	1	444.5164	1	503.1478
102	370.1561	102	444.6193	102	503.2824
204	370.2388	204	444.7192	204	503.4003
306	370.3221	306	444.8236	306	503.5138
408	370.4048	408	444.9481	408	503.6332
510	370.4862	510	445.0472	510	503.7480
612	370.5689	612	445.1494	612	503.8650

of a thick-wall closed-end cylinder subjected to internal and external pressures where:

ΔV = change of volume.

V^o = cylinder volume at zero internal and external pressure.

R_r = radius to internal wall of the cylinder.

R_J = radius to external wall of the cylinder.

P_r = pressure confined within the cylinder.

P_{Jr} = pressure acting on the external wall of the cylinder, or the jacket pressure.

σ = Poisson's ratio.

E = Young's modulus.

Equation 1 is of the form

$$\frac{\Delta V}{V^o} = kP_r + k'P_{Jr} \quad , \tag{2}$$

where

$$k = \frac{3(1-2\sigma)R_r^2 + 2(1+\sigma)R_J^2}{E(R_J^2-R_r^2)} \quad , \tag{3}$$

and

$$k' = -\frac{(5-4\sigma)R_J^2}{E(R_J^2-R_r^2)} \quad . \tag{4}$$

Equation 4 can be rearranged to give

$$E = -\frac{(5-4\sigma)R_J^2}{k'(R_J^2-R_r^2)} \quad . \tag{5}$$

A more exact form of the equation presented by Briggs and Barieau (7, p. 6, eq. 22) is

$$k' = -\frac{d\ln P_r}{dP_{J\,r}}\left(1 - \frac{d\ln Z_r}{d\ln P_r} + \frac{kP_r}{1 + kP_r + k'P_{J\,r}}\right)(1 + kP_r + k'P_{J\,r}). \qquad (6)$$

The term $\frac{d\ln P_r}{dP_{J\,r}}$ of equation 6 can be evaluated experimentally. The quantity $\frac{d\ln Z_r}{d\ln P_r}$ can be evaluated by using equation 7

$$\frac{d\ln Z_r}{d\ln P_r} = \frac{BP_r + 2CP_r^2 + 3DP_r^3 + 4EP_r^4}{1 + BP_r + CP_r^2 + DP_r^3 + EP_r^4}, \qquad (7)$$

and published compressibility data for helium (6, 8).

$Z_r = 1 + BP_r + CP_r^2 + DP_r^3 + EP_r^4$ = compressibility factor of the confined gas at P_r .

The term $(1 + kP_r + k'P_{J\,r})$ of equation 6 can be set equal to one and $\frac{kP_r}{1 + kP_r + k'P_{J\,r}}$ can be neglected without causing a significant (less than 0.1 percent) error in k'.

Reduction of the experimental observations is a bit more complicated than that of an earlier report (7) because the distortion apparatus volumes were not equivalent to the volumes V_1 or V_2 of the compressibility apparatus.

We adopt the following notation for the distortion coefficients because this notation was used in previous reports (2, 7).

α = internal-pressure distortion coefficient of volume $(V_1 + V_2)$ of the compressibility apparatus.

α' = external-pressure distortion coefficient of volume $(V_1 + V_2)$ of the compressibility apparatus.

β = internal-pressure distortion coefficient of volume V_1 of the compressibility apparatus.

β' = external-pressure distortion coefficient of volume V_1 of the compressibility apparatus.

The distortion coefficients, α, α', β, and β' are our ultimate goals. In the work of Briggs and Barieau (7), β' was measured experimentally; however, in the present investigation none of the coefficients are directly measured but they can be derived from our measurements.

Additional quantities must be defined for this work.

k_{b1} = internal-pressure distortion coefficient of the volume V_{b1}.

k'_{b1} = external-pressure distortion coefficient of the volume V_{b1}.

k_{b2} = internal-pressure distortion coefficient of the volume V_{b2}.

k'_{b2} = external-pressure distortion coefficient of the volume V_{b2}.

k'_{d1} = external-pressure distortion coefficient of the distortion apparatus when the volume V_{b1} is assembled in the jacket.

k'_{d2} = external-pressure distortion coefficient of the distortion apparatus when vessel V_{b2} is assembled in the jacket.

The coefficients k'_{d1} and k'_{d2} are experimentally determined, thus we must derive k_{b1}, k'_{b1}, k_{b2}, k'_{b2}, and ultimately α, α', β, and β' from the measured quantities. The derivation is straightforward.

Experimental values of k'_{d1} and k'_{d2}, computed from the experimental observations and equations 6 and 7, are listed in tables 3 and 4 respectively for temperatures of 0°, 25°, 50°, and 75°.

TABLE 3. - Experimental external-pressure distortion coefficients, volume V_{b1} + volume V_{fd} + volume V_{td}

Run No.	$(d\ln P_r/dP_{jr})\times 10^6$	Average $d\ln Z_r/d\ln P_r$	$k'_{d1,t}\times 10^6$, atm^{-1}	Dev. from avg $k'_{d1,t}\times 10^6$, atm^{-1}
		0° C		
(17-4-V1)-0-1	2.49778±0.01745	0.1931814	-2.01526±0.01408	+0.00990
(17-4-V1)-0-2	2.42712 ±.01714	.1721999	-2.00917 ±.01419	+.00381
(17-4-V1)-0-3	2.47432 ±.00887	.1911783	-2.00128 ±.00717	-.00408
(17-4-V1)-0-4	2.35509 ±.00708	.1517337	-1.99774 ±.00601	-.00762
(17-4-V1)-0-5	2.37470 ±.01247	.1549220	-2.00681 ±.01054	+.00145
(17-4-V1)-0-6	2.14912 ±.01794	.0684863	-2.00193 ±.01671	-.00343
Average $k'_{d1,0}$			-2.00536 ±.00467	
Average standard error of $k'_{d1,0}$			±.01145	
Standard error of a single $k'_{d1,0}$			±.00634	
		25° C		
(17-4-V1)-25-1	2.32079±0.00379	0.1398794	-1.99616±0.00326	-0.00987
(17-4-V1)-25-2	2.39238 ±.00464	.1605526	-2.00828 ±.00390	+.00225
(17-4-V1)-25-3	2.44903 ±.00728	.1782026	-2.01261 ±.00598	+.00658
(17-4-V1)-25-4	2.35808 ±.00419	.1445330	-2.01726 ±.00358	+.01123
(17-4-V1)-25-5	2.32786 ±.00576	.1426319	-1.99583 ±.00494	-.01020
Average $k'_{d1,25}$			-2.00603 ±.00433	
Average standard error of $k'_{d1,25}$			±.00433	
Standard error of a single $k'_{d1,25}$			±.00969	

TABLE 3. - Experimental external-pressure distortion coefficients, volume V_{bl} + volume V_{fd} + volume V_{td}--Continued

Run No.	$(d\ln P_r/dP_{jr})\times 10^6$	Average $d\ln Z_r/d\ln P_r$	$k'_{dl,t}\times 10^6$, atm^{-1}	Dev. from avg $k'_{dl,t}\times 10^6$, atm^{-1}
		50° C		
(17-4-V1)-50-1	2.32077±0.00482	0.1309194	-2.01694±0.00419	+0.00941
(17-4-V1)-50-2	2.37220 ±.00713	.1500108	-2.01634 ±.00606	+.00881
(17-4-V1)-50-3	2.40129 ±.00325	.1655624	-2.00373 ±.00271	-.00380
(17-4-V1)-50-4	2.36149 ±.00345	.1490928	-2.00941 ±.00294	+.00188
(17-4-V1)-50-5	2.29086 ±.00600	.1307963	-1.99122 ±.00522	-.01631
Average $k'_{dl,50}$			-2.00753 ±.00474	
Average standard error of $k'_{dl,50}$			±.00422	
Standard error of a single $k'_{dl,50}$			±.01061	
		75° C		
(17-4-V1)-75-1	2.24408±0.09035	0.1236615	-1.96657±0.07918	-0.04806
(17-4-V1)-75-2	2.39981 ±.03389	.1401877	-2.06339 ±.02914	+.04876
(17-4-V1)-75-3	2.38153 ±.00928	.1543551	-2.01393 ±.00785	-.00070
Average $k'_{dl,75}$			-2.01463 ±.02795	
Average standard error of $k'_{dl,75}$			±.03872	
Standard error of a single $k'_{dl,75}$			±.04841	

TABLE 4. - Experimental external-pressure distortion coefficients, volume V_{b2} + volume V_{fd} + volume V_{td}

Run No.	$(d\ln P_r/dP_{jr})\times10^6$	Average $d\ln Z_r/d\ln P_r$	$k'_{d2,t}\times10^6$, atm^{-1}	Dev. from avg $k'_{d2,t}\times10^6$, atm^{-1}
		0° C		
(17-4-V2)-0-1	2.25289±0.00918	0.1522721	-1.90984±0.00778	+0.01381
(17-4-V2)-0-2	2.29098 ±.00615	.1727949	-1.89511 ±.00509	-.00092
(17-4-V2)-0-3	2.33681 ±.00720	.1941399	-1.88314 ±.00580	-.01289
Average $k'_{d2,0}$			-1.89603 ±.00772	
Average standard error of $k'_{d2,0}$			±.00622	
Standard error of a single $k'_{d2,0}$			±.01337	
		25° C		
(17-4-V2)-25-1	2.18528±0.00797	0.1419355	-1.87511±0.00684	-0.00670
(17-4-V2)-25-2	2.24599 ±.00436	.1620419	-1.88205 ±.00365	+.00024
(17-4-V2)-25-3	2.30236 ±.00184	.1798521	-1.88828 ±.00151	+.00647
Average $k'_{d2,25}$			-1.88181±0.00380	
Average standard error of $k'_{d2,25}$			±.00400	
Standard error of a single $k'_{d2,25}$			±.00659	
		50° C		
(17-4-V2)-50-1	2.20576±0.00711	0.1315680	-1.91555±0.00617	+0.00010
(17-4-V2)-50-2	2.24040 ±.00406	.1504582	-1.90331 ±.00345	-.01214
(17-4-V2)-50-3	2.31841 ±.00575	.1686164	-1.92749 ±.00478	+.01204
Average $k'_{d2,50}$			-1.91545 ±.00698	
Average standard error of $k'_{d2,50}$			±.00480	
Standard error of a single $k'_{d2,50}$			±.01209	

TABLE 4. - Experimental external-pressure distortion coefficients, volume V_{b2} + volume V_{fd} + volume V_{td} --continued

Run No.	$(d\ln P_r/dP_{jr})\times 10^6$	Average $d\ln Z_r/d\ln P_r$	$k'_{d2,t}\times 10^6$, atm^{-1}	Dev. from avg $k'_{d2,t}\times 10^6$, atm^{-1}
		75° C		
(17-4-V2)-75-1	2.18497±0.00264	0.1226052	-1.91708±0.00232	+0.00244
(17-4-V2)-75-2	2.35094 ±.02757	.1420514	(-2.01699 ±.02365) 1/	+.10235
(17-4-V2)-75-3	(2.26689 ±.00584) 2/	.1564672	-1.91220 ±.00493	-.00244

Average $k'_{d2,75}$ . . . . -1.91464 ±.00257
Average standard error of $k'_{d2,75}$ . . . . ±.00363
Standard error of a single $k'_{d2,75}$ . . . . ±.00345

1/ Omitted in obtaining the average value for $k'_{d2,75}$.

2/ First pressure is omitted from the calculations.

The pertinent change of volume of the assembled compressibility apparatus, due to a change of jacket pressure, is essentially equal to the change of the jacketed volume V_{b1} or V_{b2} plus the change in volume of the jacketed nipple. We can write

$$k'_{d1} = k'_{b1} \cdot \frac{V^{o}_{b1}}{V^{o}_{d1}} + k'_{td,j} \cdot \frac{V^{o}_{td,j}}{V^{o}_{d1}} \tag{8}$$

and

$$k'_{d2} = k'_{b2} \cdot \frac{V^{o}_{b2}}{V^{o}_{d2}} + k'_{td,j} \cdot \frac{V^{o}_{td,j}}{V^{o}_{d2}} \tag{9}$$

where

$k'_{td,j}$ = external-pressure distortion coefficient of the jacketed nipple connecting the jacket cap to volume V_{b1} or V_{b2}

$$k'_{td,j} = - \frac{(5-4\sigma_{td,j})\, R^2_{j,td,j}}{E_{td,j}\,(R^2_{j,td,j} - R^2_{r,td,j})} \quad . \tag{10}$$

The high-pressure tubing is 0.25 in. od x 0.083 in. id type 304 stainless steel.

We substitute into equation 10 the numerical values,

$\sigma_{td,j}$ = 0.305 (4) at all temperatures

$R_{j,td,j}$ = 0.125 in.

$R_{r,td,j}$ = 0.0415 in.

We use the work of Briggs and Barieau (7) to obtain the values for Young's modulus as a function of temperature.

$$E_{td,j} = 1.9933 \times 10^6 \text{ atm at } 0° \text{ C}$$

$$= 1.9772 \times 10^6 \text{ atm at } 25° \text{ C}$$

$$= 1.9610 \times 10^6 \text{ atm at } 50° \text{ C}$$

$$= 1.9449 \times 10^6 \text{ atm at } 75° \text{ C}$$

Therefore

$$k'_{td,j,0} = -2.1313 \times 10^{-6} \text{ atm}^{-1}$$

$$k'_{td,j,25} = -2.1486 \times 10^{-6} \text{ atm}^{-1}$$

$$k'_{td,j,50} = -2.1664 \times 10^{-6} \text{ atm}^{-1}$$

$$k'_{td,j,75} = -2.1843 \times 10^{-6} \text{ atm}^{-1}$$

Rearranging equations 8 and 9 we obtain

$$k'_{b1} = k'_{d1} \cdot \frac{V^o_{d1}}{V^o_{b1}} - k'_{td,j} \cdot \frac{V^o_{td,j}}{V^o_{b1}} \qquad (11)$$

and

$$k'_{b2} = k'_{d2} \cdot \frac{V^o_{d2}}{V^o_{b2}} - k'_{td,j} \cdot \frac{V^o_{td,j}}{V^o_{b2}} \quad . \qquad (12)$$

Values for k'_{b1} and k'_{b2} can be calculated by using equations 11 and 12, the calculated values of $k'_{td,j}$, the known values of the volumes, and experimental values of k'_{d1} and k'_{d2}. Computed values for k'_{b1} and k'_{b2} are listed in table 5.

The distortion coefficients of table 5 are used to compute values for Young's modulus for vessels V_{b1} and V_{b2}. Equations 13 and 14 are used in the calculations.

TABLE 5. - Values for the external-pressure distortion coefficients of vessels V_{b1} and V_{b2}

Temp., °C	$k'_{b1} \times 10^6$, atm^{-1}	$k'_{b2} \times 10^6$, atm^{-1}
0	-2.0697±0.0048	-2.0129±0.0082
25	-2.0704 ±.0045	-1.9977 ±.0041
50	-2.0719 ±.0050	-2.0335 ±.0075
75	-2.0792 ±.0289	-2.0325 ±.0027

$$E_{b1} = - \frac{(5-4\sigma_{b1})R_{j,b1}^2}{k'_{b1}(R_{j,b1}^2 - R_{r,b1}^2)} \tag{13}$$

$$E_{b2} = - \frac{(5-4\sigma_{b2})R_{j,b2}^2}{k'_{b2}(R_{j,b2}^2 - R_{r,b2}^2)} \tag{14}$$

where

$$\sigma_{b1} = \sigma_{b2} = 0.272 \quad (1)$$

$$R_{j,b1} = R_{j,b2} = 1.5 \text{ in.}$$

$$R_{r,b1} = R_{r,b2} = 0.5 \text{ in.}$$

Computed values of Young's modulus are recorded in table 6 for vessels V_{b1} and V_{b2}.

We can now use the following equations to calculate the change in volume of vessels V_{b1} and V_{b2} with pressure.

$$\frac{\Delta V_{b1}}{V_{b1}^o} = \frac{3(1-2\sigma_{b1})R_{r,b1}^2 + 2(1+\sigma_{b1})R_{j,b1}^2}{E_{b1}(R_{j,b1}^2 - R_{r,b1}^2)} P_r - \frac{(5-4\sigma_{b1})R_{j,b1}^2}{E_{b1}(R_{j,b1}^2 - R_{r,b1}^2)} P_{jr} \tag{15}$$

$$\frac{\Delta V_{b2}}{V_{b2}^o} = \frac{3(1-2\sigma_{b2})R_{r,b2}^2 + 2(1+\sigma_{b2})R_{j,b2}^2}{E_{b2}(R_{j,b2}^2 - R_{r,b2}^2)} P_r - \frac{(5-4\sigma_{b2})R_{j,b2}^2}{E_{b2}(R_{j,b2}^2 - R_{r,b2}^2)} P_{jr} \tag{16}$$

By using previously listed values for the constants, we obtain:

$$\left(\frac{\Delta V_{b1}}{V_{b1}^o}\right)_0 = 1.4264 \times 10^{-6} P_r - 2.0697 \times 10^{-6} P_{jr} \tag{17}$$

$$\left(\frac{\Delta V_{b1}}{V_{b1}^o}\right)_{25} = 1.4268 \times 10^{-6} P_r - 2.0704 \times 10^{-6} P_{jr} \tag{18}$$

TABLE 6. - Values of Young's modulus of vessels V_{b1} and V_{b2}

Temp., °C	$E_{b1} \times 10^{-6}$, atm	$E_{b2} \times 10^{-6}$, atm
0	2.1264±0.0049	2.1864±0.0089
25	2.1257 ±.0046	2.2030 ±.0046
50	2.1241 ±.0052	2.1642 ±.0080
75	2.1167 ±.0290	2.1653 ±.0029

$$\left(\frac{\Delta V_{b1}}{V_{b1}^{o}}\right)_{50} = 1.4279 \times 10^{-6} P_r - 2.0719 \times 10^{-6} P_{jr} \qquad (19)$$

$$\left(\frac{\Delta V_{b1}}{V_{b1}^{o}}\right)_{75} = 1.4329 \times 10^{-6} P_r - 2.0792 \times 10^{-6} P_{jr} \qquad (20)$$

$$\left(\frac{\Delta V_{b2}}{V_{b2}^{o}}\right)_{0} = 1.3872 \times 10^{-6} P_r - 2.0129 \times 10^{-6} P_{jr} \qquad (21)$$

$$\left(\frac{\Delta V_{b2}}{V_{b2}^{o}}\right)_{25} = 1.3768 \times 10^{-6} P_r - 1.9977 \times 10^{-6} P_{jr} \qquad (22)$$

$$\left(\frac{\Delta V_{b2}}{V_{b2}^{o}}\right)_{50} = 1.4014 \times 10^{-6} P_r - 2.0335 \times 10^{-6} P_{jr} \qquad (23)$$

$$\left(\frac{\Delta V_{b2}}{V_{b2}^{o}}\right)_{75} = 1.4007 \times 10^{-6} P_r - 2.0325 \times 10^{-6} P_{jr} \qquad (24)$$

for the respective volume changes at 0°, 25°, 50°, and 75° C where the pressures P_r and P_{jr} are in atmospheres.

The change in volume of the tubing can be represented by the equation

$$\frac{\Delta V_t}{V_t^{o}} = \frac{3(1-2\sigma_t)R_{r,t}^2 + 2(1+\sigma_t)R_{j,t}^2}{E_t(R_{j,t}^2 - R_{r,t}^2)} P_r - \frac{(5-4\sigma_t)R_{j,t}^2}{E_t(R_{j,t}^2 - R_{r,t}^2)} P_{jr} \,. \qquad (25)$$

Substituting into equation 25 previously listed values for the constants, we obtain

$$\left(\frac{\Delta V_t}{V_t^{o}}\right)_{0} = 1.5443 \times 10^{-6} P_r - 2.1313 \times 10^{-6} P_{jr} \qquad (26)$$

$$\left(\frac{\Delta V_t}{V_t^{o}}\right)_{25} = 1.5569 \times 10^{-6} P_r - 2.1486 \times 10^{-6} P_{jr} \qquad (27)$$

$$\left(\frac{\Delta V_t}{V_t^o}\right)_{50} = 1.5697 \times 10^{-6} P_r - 2.1664 \times 10^{-6} P_{j\,r} \tag{28}$$

$$\left(\frac{\Delta V_t}{V_t^o}\right)_{75} = 1.5827 \times 10^{-6} P_r - 2.1843 \times 10^{-6} P_{j\,r} \quad . \tag{29}$$

We assume the fittings distort as if they were 0.25 in. od x 0.083 in. id high-pressure tubing, i.e., $\frac{\Delta V_f}{V_f^o} = \frac{\Delta V_t}{V_t^o}$. This is, of course, not the case; however, this assumption is probably not as bad as it first seems. Increasing the wall thickness of a cylinder does not change significantly the circumferential extension at the inner wall due to pressure; therefore, we can assume the volume change in the fittings can be computed as if they were tubing without significant error in the final results.

The unit change of volume V_1 of the compressibility apparatus is given by

$$\frac{\Delta V_1}{V_1^o} = \frac{\Delta V_{b1}}{V_1^o} + \frac{\Delta V_{t1}}{V_1^o} + \frac{\Delta V_{f1}}{V_1^o} = \frac{\Delta V_{b1}}{V_{b1}^o} \cdot \frac{V_{b1}^o}{V_1^o} + \frac{\Delta V_{t1}}{V_{t1}^o} \cdot \frac{V_{t1}^o}{V_1^o} + \frac{\Delta V_{f1}}{V_{f1}^o} \cdot \frac{V_{f1}^o}{V_1^o} \quad . \tag{30}$$

At 0° C we may then write

$$\begin{aligned}\left(\frac{\Delta V_1}{V_1^o}\right)_0 &= (1.4264 \times 10^{-6} P_{r-1} - 2.0697 \times 10^{-6} P_{j\,r-1})\left(\frac{4.8859}{5.0276}\right) \\ &+ (1.5443 \times 10^{-6} P_{r-1} - 2.1313 \times 10^{-6} P_{j\,r-1})\left(\frac{0.0717}{5.0276}\right) \\ &+ (1.5443 \times 10^{-6} P_{r-1} - 2.1313 \times 10^{-6} P_{j\,r-1})\left(\frac{0.0700}{5.0276}\right)\end{aligned} \tag{31}$$

or

$$\left(\frac{\Delta V_1}{V_1^o}\right)_0 = \beta_0 P_{r-1} + \beta_0' P_{j\,r-1} = 1.4297 \times 10^{-6} P_{r-1} - 2.0714 \times 10^{-6} P_{j\,r-1} \quad . \tag{32}$$

Similarly,

$$\left(\frac{\Delta V_1}{V_1^o}\right)_{25} = \beta_{25} P_{r-1} + \beta'_{25} P_{j\,r-1} = 1.4305 \times 10^{-6} P_{r-1} - 2.0726 \times 10^{-6} P_{j\,r-1} \quad (33)$$

$$\left(\frac{\Delta V_1}{V_1^o}\right)_{50} = \beta_{50} P_{r-1} + \beta'_{50} P_{j\,r-1} = 1.4319 \times 10^{-6} P_{r-1} - 2.0746 \times 10^{-6} P_{j\,r-1} \quad (34)$$

$$\left(\frac{\Delta V_1}{V_1^o}\right)_{75} = \beta_{75} P_{r-1} + \beta'_{75} P_{j\,r-1} = 1.4371 \times 10^{-6} P_{r-1} - 2.0822 \times 10^{-6} P_{j\,r-1} \cdot \quad (35)$$

The unit change of volume $(V_1 + V_2)$ of the compressibility apparatus is given by

$$\frac{\Delta(V_1+V_2)}{(V_1^o+V_2^o)} = \frac{\Delta V_{b1}}{(V_1^o+V_2^o)} + \frac{\Delta V_{b2}}{(V_1^o+V_2^o)} + \frac{\Delta(V_{t1}+V_{t2})}{(V_1^o+V_2^o)} + \frac{\Delta(V_{f1}+V_{f2})}{(V_1^o+V_2^o)}$$

$$= \frac{\Delta V_{b1}}{V_{b1}^o} \cdot \frac{V_{b1}^o}{(V_1^o+V_2^o)} + \frac{\Delta V_{b2}}{V_{b2}^o} \cdot \frac{V_{b2}^o}{(V_1^o+V_2^o)} + \frac{\Delta(V_{t1}+V_{t2})}{(V_{t1}^o+V_{t2}^o)} \cdot \frac{(V_{t1}^o+V_{t2}^o)}{(V_1^o+V_2^o)}$$

$$+ \frac{\Delta(V_{f1}+V_{f2})}{(V_{f1}^o+V_{f2}^o)} \cdot \frac{(V_{f1}^o+V_{f2}^o)}{(V_1^o+V_2^o)} \quad . \quad (36)$$

At 0° C

$$\frac{\Delta(V_1+V_2)}{(V_1^o+V_2^o)} = (1.4264 \times 10^{-6} P_r - 2.0697 \times 10^{-6} P_{j\,r})\left(\frac{4.8859}{7.6074}\right)$$

$$+ (1.3872 \times 10^{-6} P_r - 2.0129 \times 10^{-6} P_{j\,r})\left(\frac{2.5297}{7.6074}\right)$$

$$+ (1.5443 \times 10^{-6} P_r - 2.1313 \times 10^{-6} P_{j\,r})\left(\frac{0.1042}{7.6074}\right)$$

$$+ (1.5443 \times 10^{-6} P_r - 2.1313 \times 10^{-6} P_{j\,r})\left(\frac{0.0876}{7.6074}\right) \quad (37)$$

or

$$\left(\frac{\Delta(V_1+V_2)}{(V_1^0+V_2^0)}\right)_0 = \alpha_0 P_r + \alpha_0' P_{jr} = 1.4163 \times 10^{-6} P_r - 2.0524 \times 10^{-6} P_{jr} \quad . \tag{38}$$

Similarly,

$$\left(\frac{\Delta(V_1+V_2)}{(V_1^0+V_2^0)}\right)_{25} = \alpha_{25} P_r + \alpha_{25}' P_{jr} = 1.4135 \times 10^{-6} P_r - 2.0482 \times 10^{-6} P_{jr} \tag{39}$$

$$\left(\frac{\Delta(V_1+V_2)}{(V_1^0+V_2^0)}\right)_{50} = \alpha_{50} P_r + \alpha_{50}' P_{jr} = 1.4227 \times 10^{-6} P_r - 2.0615 \times 10^{-6} P_{jr} \tag{40}$$

$$\left(\frac{\Delta(V_1+V_2)}{(V_1^0+V_2^0)}\right)_{75} = \alpha_{75} P_r + \alpha_{75}' P_{jr} = 1.4260 \times 10^{-6} P_r - 2.0663 \times 10^{-6} P_{jr} \quad . \tag{41}$$

WORKING PRESSURE AND YIELD PRESSURE OF THE HIGH-PRESSURE CONTAINERS

The pressure vessels V_{b1} and V_{b2} were purchased from a commercial manufacturer of high-pressure equipment. The working pressure is 15×10^3 psi.

The containers were fully X-rayed after fabrication. The radiographs indicated complete weld penetration for V_{b1} and no weld penetration for V_{b2}; therefore, for the following calculations we assume no weld penetration.

Dimensions of the containers are shown on figures 2 and 3.

Faupel (11) presents the equation

$$P_b = \frac{2\mu_y}{\sqrt{3}} (\ln R_d)\left(2 - \frac{\mu_y}{\mu_u}\right) \tag{42}$$

for the burst pressure of a thick-wall cylinder where

P_b = burst pressure of a thick-wall cylinder.

μ_y = yield strength.

μ_u = ultimate strength.

R_d = cylinder external diameter divided by cylinder internal diameter.

The vessels were fabricated from 17-4 PH precipitation-hardening stainless steel, heat-treated in the H1150-M condition.

With

$\mu_y = 85 \times 10^3$ psi,

$\mu_u = 125 \times 10^3$ psi, and

$R_d = 2.4$,

the calculated burst pressure is

$P_b = 113 \times 10^3 \text{ psi} \approx 7.7 \times 10^3$ atm.

The vessels are to be used at working pressures to 1000 atmospheres; therefore, the safety factor is about 7.7 based upon these calculations.

Faupel (<u>11</u>) presents the equation

$$P_y = \frac{\mu_y}{\sqrt{3}} \left(\frac{R_d^2 - 1}{R_d^2} \right) \tag{43}$$

for the elastic breakdown pressure of a heavy wall cylinder, where

P_y = yield pressure.

Substituting into equation (43) we obtain

$P_y = 40.6 \times 10^3 \text{ psi} \approx 2.76 \times 10^3$ atm.

The pressure containers were tested to 22.5×10^3 psi or 1.5 times the design working pressure of 15×10^3 psi; therefore, the forces were below the proportional limit of the material of construction. We assume that there was no permanent distortion due to the pressure test.

DISCUSSION

The values of Young's modulus for 17-4 PH stainless steel, computed from our experimental measurements are larger than the value reported in the literature (1)[3/]. Also, Young's modulus for V_{b2} is larger than that

---

3/ For this comparison, we assume that the value (28.5×10^6 psi) of Young's modulus reported for condition H900 is applicable for all hardened conditions.

---

for V_{b1}. This means there was less distortion of the vessels than one would calculate from the distortion equations and literature value for Young's modulus.

The decrease in distortion and attendant increase in the computed values for Young's modulus are, no doubt, due to end effects, as the distortion equations do not correct for this. Computed values for Young's modulus are larger for vessel V_{b2} than for V_{b1} because V_{b2} is shorter than V_{b1}, and end effects would be expected to be more pronounced in V_{b2}.

Our experimentally determined value for Young's modulus, for room temperature, for V_{b1} is about 10 percent higher than the literature value and about 14 percent higher for V_{b2}. The significant aspect of these measurements is that an error of 10 to 14 percent would be introduced into the distortion coefficients for our particular pressure vessels if end effects were neglected.

The method of least squares was used to fit the data of table 6 to the equations

$$E_{b1} = (2.12783 \pm 0.002085) \times 10^6 - (1.228 \pm 0.446) \times 10^2 t \tag{44}$$

$$E_{b2} = (2.19504 \pm 0.01341) \times 10^6 - (4.084 \pm 2.868) \times 10^2 t \tag{45}$$

where t = temperature, °C.

Equations 44 and 45 indicate a small decrease in Young's modulus with increasing temperature, but the observed effect of temperature was less than expected.

We believe the distortion coefficients of volumes V_1 and (V_1+V_2) of the compressibility apparatus are known to about one percent. An error of about one percent in the distortion coefficients would cause an error of about 0.0015 percent in the calculated compressibility factor for helium at 0° C and 1000 atmospheres pressure.

REFERENCES

1. Armco Steel Corporation. Armco 17-4 PH Precipitation-Hardening Stainless Steel Bar and Wire. Product Data, No. S-6a, 16pp.

2. Barieau, Robert E., and B. J. Dalton. A Method for Treating PVT Data From a Burnett Compressibility Apparatus. BuMines Rept. of Inv. 7020, September 1967, 34 pp.

3. Bartlett, Edward P. The Compressibility Isotherms of Hydrogen, Nitrogen and Mixtures of These Gases at 0° and Pressures to 1000 Atmospheres. J. Am. Chem. Soc., v. 49, No. 3, March 1927, p. 691.

4. Baumeister, Theodore, Ed. Mechanical Engineers' Handbook. McGraw-Hill Book Company, Inc., New York, 1958, p. 5-6.

5. Blancett, Allen Leroy. Volumetric Behavior of Helium-Argon Mixtures at High Pressure and Moderate Temperature. Ph.D. Thesis, Univ. of Oklahoma, 1966, 228 pp. Univ. Microfilms, Inc., Ann Arbor, Mich., Order No. 66-14,196.

6. Briggs, Ted C. Compressibility Data for Helium Over the Temperature Range -5° to 80° C and at Pressures to 800 Atmospheres. Helium Research Center Internal Report 120, August 1969, 57 pp. On file at the Helium Research Center, Bureau of Mines, Amarillo, Tex.

7. Briggs, Ted C., and Robert E. Barieau. Elastic Pressure Distortion of the Volumes of a Burnett Compressibility Apparatus Over the Temperature Range 0° to 80° C. BuMines Rept. of Inv. 7136, June 1968, 32 pp.

8. Briggs, Ted C., B. J. Dalton, and Robert E. Barieau. Compressibility Data for Helium at 0° C and Pressures to 800 Atmospheres. BuMines Rept. of Inv. 7287, August 1969, 54 pp.

9. Burnett, E. S. Compressibility Determinations Without Volume Measurements. J. Appl. Mech., v. 3, No. 4, December 1936, pp. A136-A140.

10. Canfield, Frank B., Jr. The Compressibility Factors and Second Virial Coefficients for Helium-Nitrogen Mixtures at Low Temperature and High Pressure. Ph.D. Thesis, Rice Univ., May 1962, 321 pp.

11. Faupel, Joseph H. Engineering Design. John Wiley and Sons, Inc., New York, 1964, 980 pp.

12. Love, A. E. H. A Treatise on the Mathematical Theory of Elasticity. Dover Publications, New York, 4th. ed., 1927, 643.

13. Mueller, William H. Volumetric Properties of Gases at Low Temperatures by the Burnett Method. Ph.D. Thesis, Rice Univ., December 1959, 138 pp.

14. Newitt, Dudley M. The Design of High Pressure Plant and the Properties of Fluids at High Pressures. Oxford University Press, London, England, 1940, 491 pp.